# An Application of Inverse Functions: Displacement of a Spring

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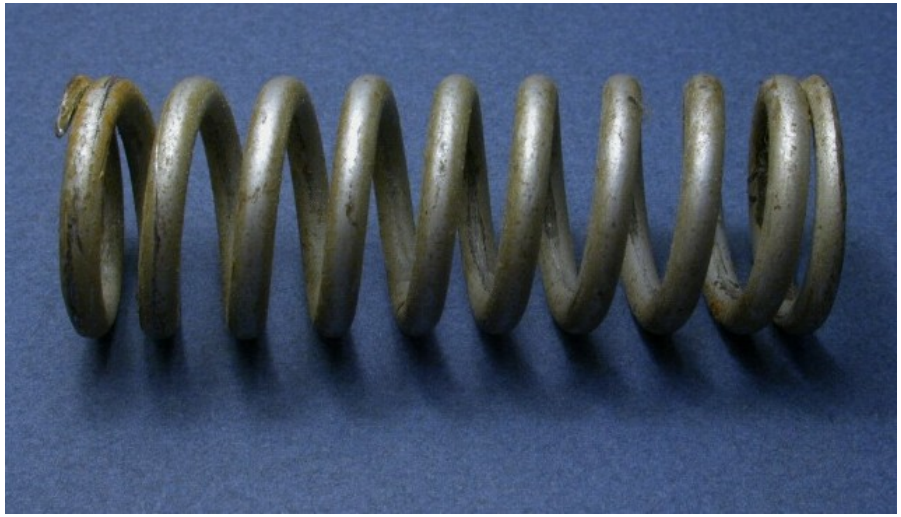
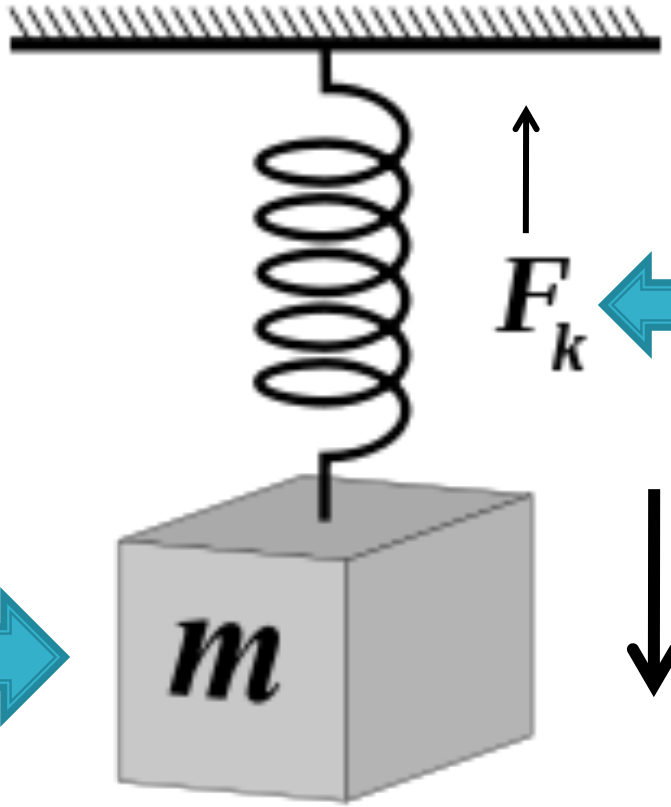
# Hooke's Law

- In mechanics, and physics, **Hooke's law** of elasticity is an approximation that states that the extension (displacement) of a spring is in direct proportion with the load (force) added.






**Force** exerted by the spring

$d$ = displacement

Load with mass ($m$)

**Force exerted on the spring (weight of mass)**

# Mathematical Statement of Hooke's Law

- Mathematically, Hooke's law states that

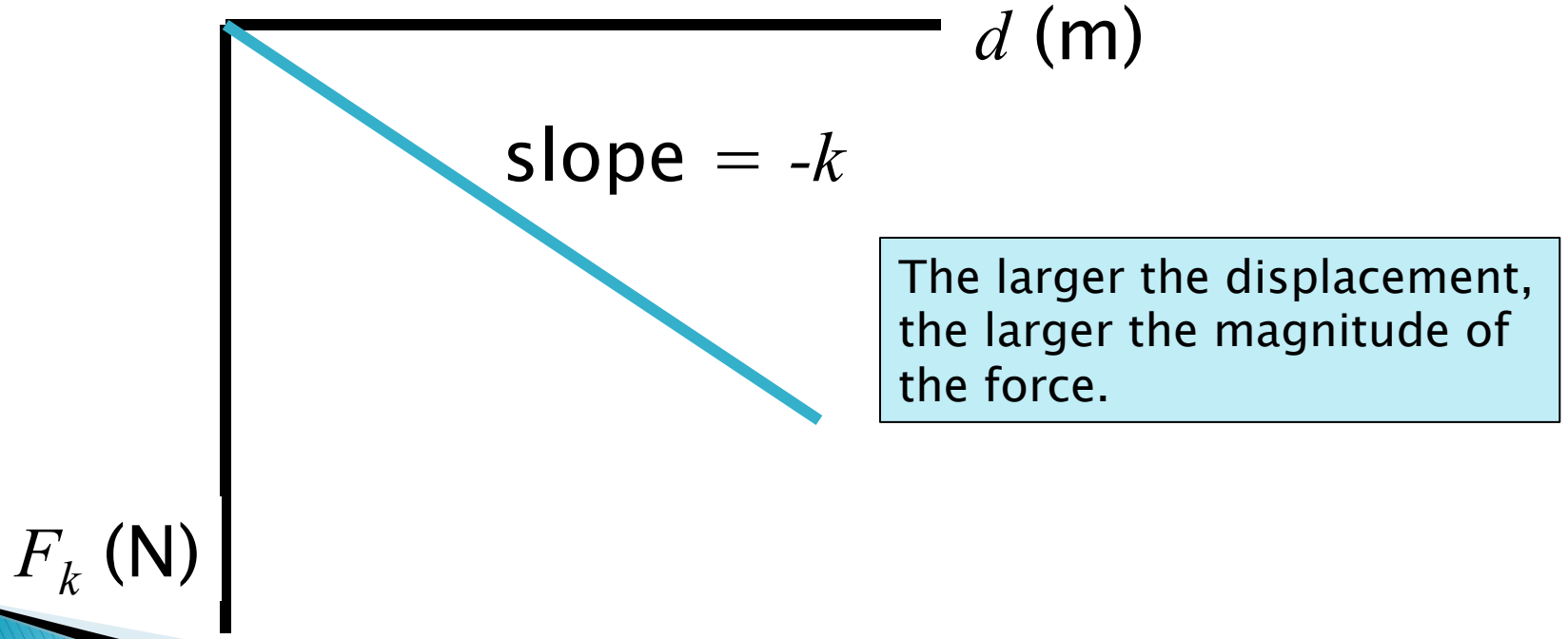
$$F_k = -k\,d$$

where:

- $d$ is the **displacement** of the end of the spring from its **equilibrium** position (in **SI** units: "m");
- $F_k$ is the restoring force exerted by the spring (in **SI** units: Newtons (N));
- and $k$ is the ***force constant*** (or ***spring constant***) (in **SI** units: "$N \cdot m^{-1}$")

# Plot of Force vs. Displacement

- The behavior of the force is said to be ***linear***.
- The plot of force ($F$) and displacement ($x$) appears as a ***straight line***.




The larger the displacement, the larger the magnitude of the force.

# Why is the Force Negative?

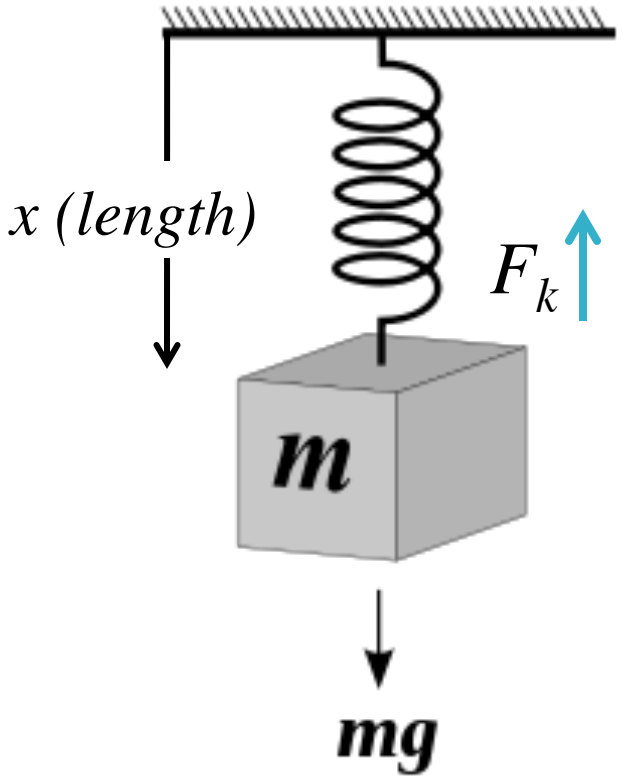
- There is a negative sign on the right hand side of the equation because the restoring force always acts in the opposite direction of the displacement (for example, when a spring is stretched downward, it pulls upward.

- Let x represent the length of a spring in cm and m the mass of an object in kg
- When no mass is attached to the spring, x = 50.0.
- The force constant is $k = 21.8$ N/cm.
- Express $x$ as a function of $m$

$$x = 50 + \frac{mg}{k}$$

$$x = 50 + m\left(\frac{9.8}{21.8}\right)$$

$$\boxed{x = 50 + 0.450m}$$

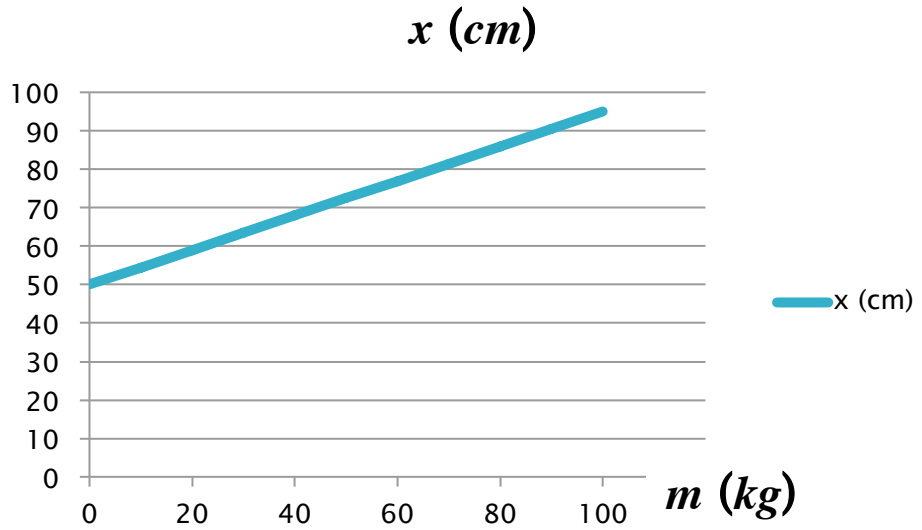


# Forward Function

- Using Hooke's Law, we can express $x$ as a function of $m$.

$$x = f(m) = 50 + 0.450\,m \qquad m \geq 0$$

| *m (kg)* | *x (cm)* |
|---|---|
| 0 | 50 |
| 10 | 54.5 |
| 20 | 59 |
| 30 | 63.5 |
| 40 | 68 |
| 50 | 72.5 |
| 60 | 77 |
| 70 | 81.5 |
| 80 | 86 |
| 90 | 90.5 |
| 100 | 95 |



# Inverse Function

- The challenge now is to find a “inverse” function that expresses m in terms of x.

$$m = f^{-1}(x)$$

# Deriving the Inverse Function

$$x = f(m) = 50 + 0.450\,m$$

- Since $x$ is an increasing function of $m$, $f$ is invertible. Let's solve the equation for $m$.

$$x - 50 = 0.450\,m$$

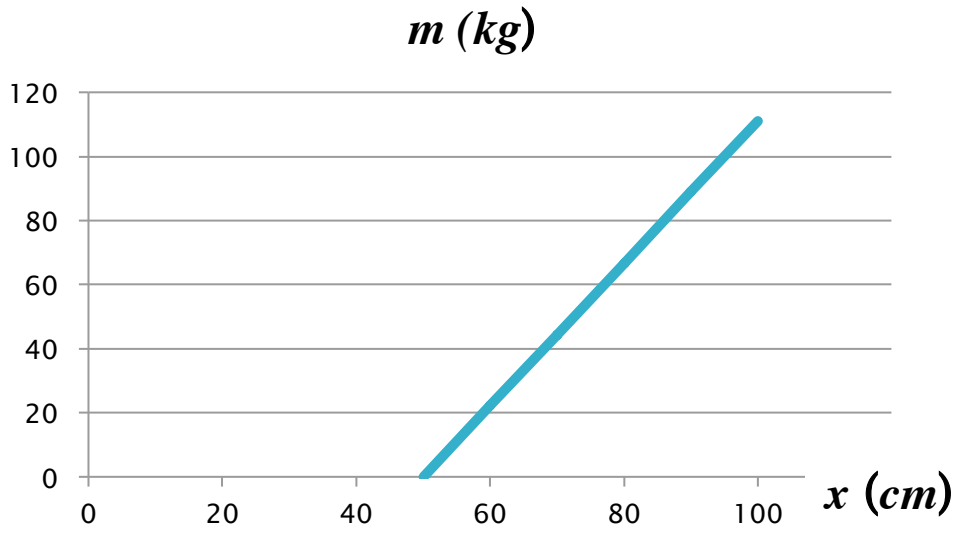
$$\frac{x - 50}{0.45} = m$$

$$m = f^{-1}(x) = \frac{x}{0.45} - 111.1$$

# Inverse Function

$$m = f^{-1}(x) = \frac{x}{0.45} - 111.1 \qquad x \geq 50$$

| *x (cm)* | *m (kg)* |
|---|---|
| 50 | 0 |
| 60 | 22 |
| 70 | 44 |
| 80 | 67 |
| 90 | 89 |
| 100 | 111 |



# Summary

- ***Hooke's Law*** is a physical law that relates the displacement of a spring to the force that is applied.
- The expansion and force of a spring are related by a constant known as the ***spring constant***.
- The length of a spring can be expressed as a function of the mass attached to it.
- An inverse function can be found that expresses the mass attached in terms of the length.